ANALYSIS OF A CYCLOTRON TYPE TUBE

by

Paul G. Baird

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ENGINEERING EXPERIMENT STATION
UNIVERSITY OF COLORADO
BOULDER, COLORADO

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ABSTRACT

Using a combination of analytical and numerical methods an analysis is carried out on a microwave detector of cyclotron type. Curves of signal current versus input power are determined for grids with square openings. Several different grid sizes are considered. Results are given in normalized form so as to be applicable to a general tube.

It is found that for low input powers output signal current varies linearly with input power. Changes in total signal current of the order of ten per cent are indicated for one tenth milliwatt input. An optimum grid size is indicated. Considering grids larger than the optimum, the smaller grids are more sensitive but have saturated output for lower input powers. Frequencies other than the natural frequency of the tube are sharply discriminated against. Electron transit time is quite important in determining tube characteristics.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

Figure Number		Follows Page

Chapter I

INTRODUCTION

1.1 The Tube

A rectangular wave guide is used in the TE_{10} mode. A wire mesh forms one edge. The wire mesh is backed up by the cathode. The opposite edge of the wave guide is a fine grid, whose openings are approximately 10 mils across. Outside the wave guide beyond the grid is a collector plate. The collector plate catches those electrons which are emitted from the cathode but not captured by the grid. The entire tube is immersed in a uniform magnetic field whose direction is from cathode to collector plate. The wave guide is operated at a D.C. potential of one or two volts above the cathode and the collector is operated at a slightly higher potential.

1.2 Operation of the Tube

The function of the tube is to detect the presence of high frequency electromagnetic waves. For the tube to be considered the frequency will be around 3000 megacycles.

Electrons leaving the cathode spiral around the magnetic flux lines as they cross the guide. While the radius of the spiral depends upon the sidewise component of velocity of the electrons, the period of the revolution depends only upon the strength of the magnetic field. By proper adjustment of the magnetic flux density, then, this period may be

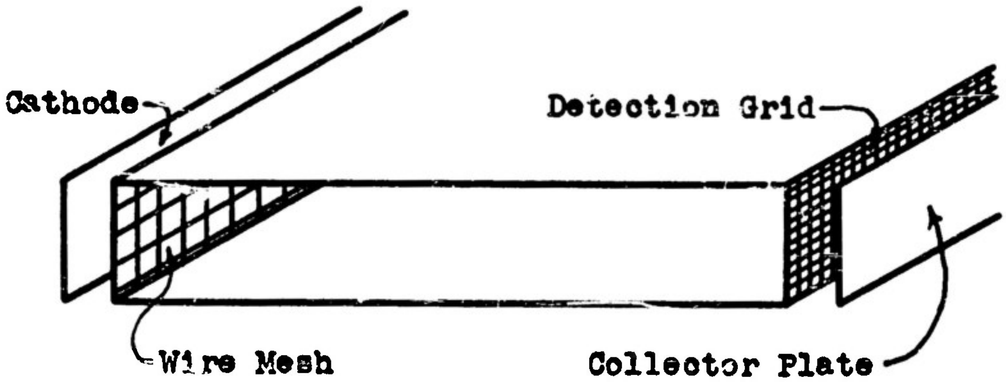


Figure 1 -- The Cyclotron Resonance Tube

made the same as the period of the wave which the tube is designed to detect. If an electromagnetic wave is present in the wave guide while electrons are crossing, there will be interaction between the electrons and this wave. Suppose this interaction should cause a net change in radius of the electrons. It seems reasonable to assume that the grid might capture more or less electrons than it did in the absence of the wave. Thus the grid current and collector current would undergo variations dependent upon the strength of the electromagnetic wave.

1.3 The Problem

The problem to be considered is an analysis of the tube characteristics. Let us determine the relation between collector current and microwave power input. Then let us determine the effect of changing the size of the grid openings or the transit time (transit time is changed by changing accelerating voltage or width of the wave guide). Also let us determine the magnitude of discrimination against neighboring frequencies.

1.4 Assumptions

It is inevitable in a problem of this sort that some assumptions be made. Whether or not the assumptions involved in this analysis will yield accurate results will have to be determined by comparing the results to be obtained with experimental results.

It will be assumed that the electrons enter the wave guide with a distribution of sidewise velocities which is the same as the distribution at the cathode. Further it will be assumed that the transit velocity (in the direction from cathode to collector) is constant except perhaps at the edges of the guide. With the TE_{10} mode the distribution of electric field intensity across the guide is a half sinusoid and so zero at the edges. Thus it is of no great consequence whether or not the transit velocity is constant at the edges as far as interaction between the electromagnetic wave and the electrons is concerned.

It will also be assumed that there exists in the neighborhood of the grid no appreciable component of electrostatic field tending to attract electrons to the grid from the openings in the grid. Also the grid will have a front-to-back dimension which equals or exceeds v/f where v is the velocity corresponding to the accelerating voltage and f is the frequency of the wave to be detected. Any electrons that have sufficient radius of spiral to spiral into the grid vanes will be assumed to be captured by the grid.

1.5 Method of Attack

Consider first those electrons whose axis of spiral intersects the grid vanes. Those electrons are sure to be captured by the grid, without regard to the radius of spiral. If the grid structure occupies, say, fifteen per cent of the space in the grid region, then at least fifteen per cent of the total cathode current will go to the grid. There will be more grid current than this however, even with quiescent conditions. Electrons are emitted from the cathode with a distribution of velocities. A component of these initial velocities will be parallel to the cathode surface and hence effective in producing the circular motion about the magnetic flux lines. Consider then an electron whose axis of spiral passes through a grid opening. This electron may or may not be captured by the grid. We will consider it to be captured by the grid if the distance between the axis of the spiral and the nearest grid vane is less than the radius of the spiral.

The grid openings will be divided into bands such that for a given band any part is approximately the same distance from the closest grid vane. Electrons belonging to a particular band and having a radius exceeding the average distance of this band from the grid will be assumed to be captured. The remainder of the electrons belonging to this particular band will go on to the collector plate. Equations will be derived so that it can be determined what portion of the electrons belonging to a particular band have a radius exceeding the distance of this band from the grid. When this is done it will be found that the change in radius for a particular electron depends upon its initial radius, the phase relative to the phase of the incoming wave, the transit time, and the input power.

The initial quantity of electrons with sidewise speeds lying between v_t and $v_t + dv_t$ is $f(v_t)\,dv_t$ where[1]

$$f(v_t) = A v_t \epsilon^{-\frac{m}{2KT} v_t^2} . \tag{1}$$

For convenience we choose A so that

$$\int_0^\infty f(v_t)\,dv_t = 1 . \tag{2}$$

A = $\frac{m}{KT}$ satisfies this requirement.

Hence $$f(v_t) = \frac{m v_t}{KT} \epsilon^{-\frac{m}{2KT} v_t^2} . \tag{3}$$

m = mass of an electron in kg

v_t = speed in meters per second

K = Boltzman's constant

T = temperature of the cathode in degrees kelvin

By our choice for the constant, A, we can consider the total current under consideration (the current approaching the grid openings) to be unity. It will be found more convenient however, to talk in terms of radius rather than speed. Therefore let us set up a new function $g(r_0)$ which will give us the initial distribution in terms of initial radius, r_0 . To do this we note that

$$r_0 = v_t / \omega_0 \qquad \text{where} \qquad \omega_0 = 2\pi f_0 .$$

f_0 = natural frequency due to the magnetic field.

[1] E. H. Kennard, Kinetic Theory of Gases, (New York: McGraw Hill Book Company, 1938), page 47

To obtain the function $g(r_0)$, then, we replace v_t by $\omega_0 r_0$ in the function $f(v_t)$ and multiply the result by ω_0 since $dv_t = \omega_0 \, dr_0$ and we require

$$\int_0^\infty g(r_0)\, dr_0 = 1 \quad . \tag{4}$$

$$g(r_0) = \frac{m\omega_0^2 r_0}{KT} e^{-\frac{m\omega_0^2}{2KT} r_0^2} \tag{5}$$

For future reference let us also note that

$$\int_{r_i}^\infty g(r_0)\, dr_0 = e^{-\frac{m\omega_0^2}{2KT} r_i^2} . \tag{6}$$

By using this last equation we can talk about what part of the cathode current will go to the grid under quiescent conditions. The grid openings will be divided into bands as previously described. Some portion of the cathode current will belong to each of these bands. This portion will be directly proportional to the area of the band under consideration (constant current density is assumed). Since we are taking the total current approaching the grid openings to be unity the portion belonging to any band is given by the ratio of the area of one of these bands to the area of a grid opening. Suppose a particular band has an area A_i. The current belonging to this band is then given by A_i/A_t where A_t is the area of the grid opening.

The band under consideration, the i^{th}, is located at an average distance, r_i, from the grid vanes. From equation (6)

$$\int_{r_i}^{\infty} g(r_0)\, dr_0 = \epsilon^{-\frac{m\omega_0^2}{2KT} r_i^2} .$$

Thus of the current A_i/A_t a portion $(A_i/A_t)\,\epsilon^{-\frac{m\omega_0^2}{2KT} r_i^2}$ is made up of electrons whose radii exceed r_i. This portion of the current (belonging to the band A_i) goes to the grid. Summing up for all the bands, $(p+1)$, we get for the quiescent grid current

$$I_{G0} = \sum_{i=0}^{p} \frac{A_i}{A_t}\, \epsilon^{-\frac{m\omega_0^2}{2KT} r_i^2} . \tag{7}$$

This excludes, of course, the current captured because of the finite thickness of the grid vanes.

Notice that this procedure makes the total cathode current greater than unity by an amount that depends upon the cross sectional area of the grid. Thus if the cross sectional area of the grid were fifteen per cent of the total area then the total cathode current would be $1 + \frac{.15}{1-.15}$.

Let us consider the calculation of the grid current when an input signal is present. We have established that the portion of the electrons which have a final radius exceeding r_i is given by $\epsilon^{-\frac{m\omega_0^2}{2KT} r_i^2}$ for quiescent conditions. Suppose that in some way we could find the portion of electrons having radii exceeding any given r_i for a given input power and frequency. Then let differences be taken between these values and the values given by $\epsilon^{-\frac{m\omega_0^2}{2KT}r_i^2}$. Let these differences be denoted δ_i, where δ_i tells us the increase in the portion of the electrons that have radii exceeding r_i. By using reasoning similar to what we used to calculate the grid current for quiescent conditions we get for the increase in grid current

$$\Delta I_G = \sum_{i=0}^{p} \frac{A_i}{A_t} \delta_i \quad . \tag{8}$$

The problem remains to determine what portion of the electrons have radii exceeding a given r_i after passage across the wave guide. This will be a topic for continued discussion.

In order to solve this problem let us determine what change in sidewise speed takes place as an electron crosses the wave guide. This change in speed divided by ω_0 will give us the change in radius. We know initially what the various radii are and what portion of the electrons lie between any two given radii. This information we have from the distribution function $g(r/r_0)$. If we can establish a formula that tells us what the change in radius is for a given set of initial conditions, then we can establish a new distribution function. If this can not be done conveniently analytically then it can be done numerically. Let groups be formed, each group containing electrons with nearly the same radii. Since we know initially how many electrons belong in each group, we can re-distribute these electrons into their proper final group, provided we know what change in radius has taken place. Any electrons which don't fall clearly into one of these final groups can be divided between two adjacent groups, interpolating on the basis of the change in radius.

We can then say what portion of the electrons have radii exceeding a given R_i . Add up all groups containing electrons with radii exceeding R_i and this will be the answer.

The method of attack has now been described. Let us consider the problem of finding the change in sidewise speed.

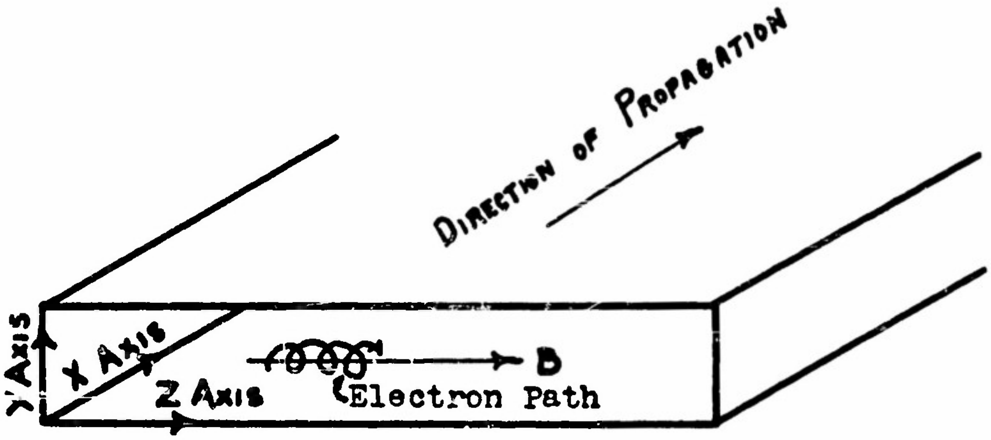
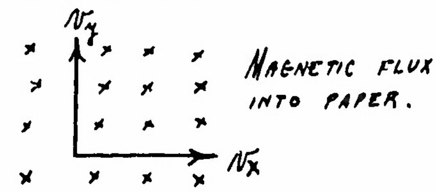


Figure 2 -- Choice of Axes

Chapter II

THE DIFFERENTIAL EQUATIONS

2.1 Setting up the Differential Equations



Let us set up the differential equations from which we can determine the final sidewise speed (and hence radius). We start with

$$F_x = Bev_y \quad \text{and} \quad F_y = -Bev_x - E_y e .$$

F_x = x-component of force on an electron, newtons

F_y = y-component of force on an electron, newtons

e = charge of an electron, coulombs

B = magnetic flux density, webers per square meter

E_y = electric field intensity in y direction due to TE_{10} wave

m = mass of an electron, kilograms

a_x = acceleration in x direction

a_y = acceleration in y direction

Hence $a_x = \dfrac{Bev_y}{m}$ and $a_y = -\dfrac{Bev_x}{m} - \dfrac{E_y e}{m}$.

Since $a_x = \dfrac{d}{dt} v_x$ and $a_y = \dfrac{d}{dt} v_y$ we write

$$\frac{dv_x}{dt} = \frac{Bev_y}{m} \quad \text{and} \quad \frac{dv_y}{dt} = -\frac{Bev_x}{m} - \frac{E_y e}{m} . \tag{9}$$

By solving equations (9) for v_x and v_y we get

$$v_y = \frac{m}{Be}\frac{dv_x}{dt} = \frac{1}{\omega_0}\frac{dv_x}{dt} \qquad \left(\omega_0 = \frac{Be}{m}\right)$$

and (10)

$$v_x = -\left(\frac{E_y}{B} + \frac{1}{\omega_0}\frac{dv_y}{dt}\right).$$

By differentiating the first of equations (10) we get

$$\frac{dv_y}{dt} = \frac{1}{\omega_0}\frac{d^2v_x}{dt^2}. \tag{11}$$

By differentiating the second we get

$$\frac{dv_x}{dt} = -\left(\frac{1}{B}\frac{dE_y}{dt} + \frac{1}{\omega_0}\frac{d^2v_y}{dt^2}\right). \tag{12}$$

We can obtain equations in v_x alone and v_y alone from equations (11) and (12).

$$v_x = -\left(\frac{E_y}{B} + \frac{1}{\omega_0^2}\frac{d^2v_x}{dt^2}\right) \tag{13}$$

$$v_y = -\left(\frac{1}{B\omega_0}\frac{dE_y}{dt} + \frac{1}{\omega_0^2}\frac{d^2v_y}{dt^2}\right) \tag{14}$$

These can be written as follows:

$$(D^2+\omega_0^2)v_x = -\frac{\omega_0^2}{B}E_y, \tag{15}$$

$$(D^2+\omega_0^2)v_y = -\frac{\omega_0}{B}\frac{dE_y}{dt}. \tag{16}$$

The expression for E_y (TE_{10} mode) is

$$E_y = E_0 \sin\left(\frac{\pi z}{z_0}\right) \sin \omega t \tag{17}$$

where z_0 is the width of the guide.

If we assume a uniform velocity of transit across the guide in time $t = \tau$ then $z = \frac{z_0 t}{\tau}$ at any time t.

Hence $$E_y = E_0 \sin\left(\frac{\pi t}{\tau}\right) \sin \omega t \,. \tag{18}$$

Also $$\frac{dE_y}{dt} = E_0 \omega \sin\left(\frac{\pi t}{\tau}\right) \cos \omega t + \frac{E_0 \pi}{\tau} \cos\left(\frac{\pi t}{\tau}\right) \sin \omega t \,. \tag{19}$$

We are not necessarily restricting ω to be the same as ω_0. If we expand the product of the two sine terms in the expression for E_y and the product of the sine and cosine terms in the expression for $\frac{dE_y}{dt}$ we obtain the following results:

$$E_y = \frac{E_0}{2} \cos\left(\left[\omega - \frac{\pi}{\tau}\right]t\right) - \frac{E_0}{2} \cos\left(\left[\omega + \frac{\pi}{\tau}\right]t\right) ; \tag{20}$$

$$\frac{dE_y}{dt} = \frac{E_0 \omega}{2} \sin\left(\left[\omega + \frac{\pi}{\tau}\right]t\right) - \frac{E_0 \omega}{2} \sin\left(\left[\omega - \frac{\pi}{\tau}\right]t\right) \tag{21}$$

$$+ \frac{E_0 \pi}{2\tau} \sin\left(\left[\omega + \frac{\pi}{\tau}\right]t\right) + \frac{E_0 \pi}{2\tau} \sin\left(\left[\omega - \frac{\pi}{\tau}\right]t\right) .$$

Using these last expressions we re-write equations (15) and (16) as

$$(D^2+\omega_0^2)\,v_x = \frac{E_0\omega_0^2}{2B}\cos\left(\left[\omega+\frac{\pi}{\tau}\right]t\right) - \frac{E_0\omega_0^2}{2B}\cos\left(\left[\omega-\frac{\pi}{\tau}\right]t\right) \tag{22}$$

and

$$(D^2+\omega_0^2)\,v_y = -\frac{\omega_0 E_0(\omega\tau+\pi)}{2B\tau}\sin\left(\left[\omega+\frac{\pi}{\tau}\right]t\right) + \frac{\omega_0 E_0(\omega\tau-\pi)}{2B\tau}\sin\left(\left[\omega-\frac{\pi}{\tau}\right]t\right). \tag{23}$$

2.2 Solution of the Differential Equations

For the particular solution of equation (23)

$$v_y = C_1\sin\left(\left[\omega+\frac{\pi}{\tau}\right]t\right) + C_2\cos\left(\left[\omega+\frac{\pi}{\tau}\right]t\right) + C_3\sin\left(\left[\omega-\frac{\pi}{\tau}\right]t\right) + C_4\cos\left(\left[\omega-\frac{\pi}{\tau}\right]t\right). \tag{24}$$

By differentiating this expression and substituting into equation (23) we find

$$\begin{aligned} C_1 &= -\frac{\omega_0 E_0(\omega\tau+\pi)}{2B\tau\left[\omega_0^2-\omega^2-\frac{2\pi\omega\tau+\pi^2}{\tau^2}\right]}, \\ C_2 &= 0, \\ C_3 &= \frac{\omega_0 E_0(\omega\tau-\pi)}{2B\tau\left[\omega_0^2-\omega^2+\frac{2\pi\omega\tau-\pi^2}{\tau^2}\right]}, \text{ and} \\ C_4 &= 0. \end{aligned} \tag{25}$$

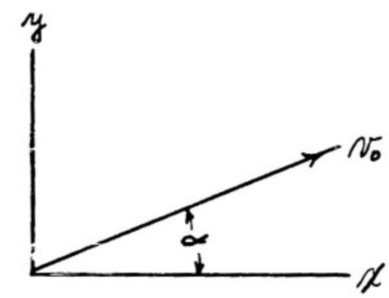
The complementary solution is

$$v_y = C_5 \sin \omega_0 t + C_6 \cos \omega_0 t \ . \tag{26}$$

The boundary conditions are

$$v_y = v_0 \sin \alpha \text{ at } t = 0 \quad (0 \leq \alpha < 360^\circ) \quad \text{and}$$

$$\frac{dv_y}{dt} = -\frac{Bev_0}{m}\cos\alpha = -v_0\omega_0 \cos\alpha \text{ at } t = 0 \ . \tag{27}$$



It follows that

$$c_5 = -v_0\cos\alpha - \frac{C_1}{\omega_0}\left(\frac{\omega\tau+\pi}{\tau}\right) - \frac{C_3}{\omega_0}\left(\frac{\omega\tau-\pi}{\tau}\right) \quad \text{and}$$

$$c_6 = v_0 \sin\alpha \ . \tag{28}$$

The complete solution is

$$v_y = \left[-v_0\cos\alpha + \frac{E_0(\omega\tau+\pi)^2}{2B\tau^2\left[\omega_0^2-\omega^2-\frac{2\pi\omega\tau+\pi^2}{\tau^2}\right]} - \frac{E_0(\omega\tau-\pi)^2}{2B\tau^2\left[\omega_0^2-\omega^2+\frac{2\pi\omega\tau-\pi^2}{\tau^2}\right]}\right]\sin\omega_0 t$$

$$+ v_0\sin\alpha\cos\omega_0 t - \left[\frac{\omega_0 E_0(\omega\tau+\pi)}{2B\tau\left[\omega_0^2-\omega^2-\frac{2\pi\omega\tau+\pi^2}{\tau^2}\right]}\right]\sin\left(\left[\omega+\frac{\pi}{\tau}\right]t\right) \tag{29}$$

$$+ \left[\frac{\omega_0 E_0(\omega\tau-\pi)}{2B\tau\left[\omega_0^2-\omega^2+\frac{2\pi\omega\tau-\pi^2}{\tau^2}\right]}\right]\sin\left(\left[\omega-\frac{\pi}{\tau}\right]t\right) .$$

By a similar process we can solve equation (22). The boundary conditions are

$$v_x = v_0 \cos\alpha \quad \text{at } t = 0 \quad \text{and}$$

$$\frac{dv_x}{dt} = \omega_0 v_0 \sin\alpha \quad \text{at } t = 0 . \tag{30}$$

The complete solution is

$$v_x = v_0 \sin\alpha \sin\omega_0 t$$
$$+ \left[v_0\cos\alpha - \frac{E_0\omega_0^2}{2B\left[\omega_0^2 - \omega^2 - \frac{2\pi\omega\mathcal{T}+\pi^2}{\mathcal{T}^2}\right]} + \frac{E_0\omega_0^2}{2B\left[\omega_0^2 - \omega^2 + \frac{2\pi\omega\mathcal{T}-\pi^2}{\mathcal{T}^2}\right]} \right] \cos\omega_0 t$$
$$+ \left[\frac{E_0\omega_0^2}{2B\left[\omega_0^2 - \omega^2 - \frac{2\pi\omega\mathcal{T}+\pi^2}{\mathcal{T}^2}\right]} \right] \cos\left(\left[\omega + \frac{\pi}{\mathcal{T}}\right]t\right) \tag{31}$$
$$- \left[\frac{E_0\omega_0^2}{2B\left[\omega_0^2 - \omega^2 + \frac{2\pi\omega\mathcal{T}-\pi^2}{\mathcal{T}^2}\right]} \right] \cos\left(\left[\omega - \frac{\pi}{\mathcal{T}}\right]t\right) .$$

In our problem the electrons will pass through several hundred cycles before traversing the wave guide. It should not make much difference in the final answer, then, if we were to add or subtract from the final electron speed a portion equal to that gained or lost in a fraction of one cycle. This will be the justification for adjusting the transit time, $\mathcal{T}$, slightly in order to simplify the equations. We are interested in the final speed at time $t = \mathcal{T}$.

Let us choose $\mathcal{T}$ so that $\sin \omega\mathcal{T} = 0$. Then if we consider only those frequencies, $\frac{\omega}{2\pi}$, for which $\sin \omega\mathcal{T} = 0$, we must have $\omega\mathcal{T} = k\pi$ (k an integer). For the special case $\omega = \omega_0$

$$\omega_0 \mathcal{T} = n\pi .$$

From the last two equations we have

$$\omega = \frac{k}{n}\omega_0 . \tag{32}$$

By restricting our discussion to those frequencies for which $\sin \omega\mathcal{T} = 0$ we obtain for v_x and v_y at time $t = \mathcal{T}$ the following equations:

$$v_x = (-1)^n \left[v_0 \cos\alpha - \frac{E_0 \omega_0^2}{2B\left[\omega_0^2 - \omega^2 - \frac{2\pi\omega\mathcal{T} + \pi^2}{\mathcal{T}^2}\right]} + \frac{E_0 \omega_0^2}{2B\left[\omega_0^2 - \omega^2 + \frac{2\pi\omega\mathcal{T} - \pi^2}{\mathcal{T}^2}\right]} \right]$$
$$+ (-1)^k \left[\frac{E_0 \omega_0^2}{2B\left[\omega_0^2 - \omega^2 + \frac{2\pi\omega\mathcal{T} - \pi^2}{\mathcal{T}^2}\right]} - \frac{E_0 \omega_0^2}{2B\left[\omega_0^2 - \omega^2 - \frac{2\pi\omega\mathcal{T} + \pi^2}{\mathcal{T}^2}\right]} \right] \tag{33}$$

and

$$v_y = (-1)^n v_0 \sin\alpha . \tag{34}$$

Since $2\pi\omega J$ will be around a thousand times the size of π^2 let us neglect π^2 by comparison and further simplify the equations. Also let us eliminate $\mathcal{B}$ by using the relation $\mathcal{B} = \frac{\omega_0 m}{c}$ to obtain the following equations:

$$N_x = (-1)^n\left[N_0\cos\alpha - \frac{E_0 e\,\omega_0}{2m\left[\omega_0^2-\omega^2-\frac{2\pi\omega}{J}\right]} + \frac{E_0 e\,\omega_0}{2m\left[\omega_0^2-\omega^2+\frac{2\pi\omega}{J}\right]}\right]$$

$$+(-1)^k\left[\frac{E_0 e\,\omega_0}{2m\left[\omega_0^2-\omega^2+\frac{2\pi\omega}{J}\right]} - \frac{E_0 e\,\omega_0}{2m\left[\omega_0^2-\omega^2-\frac{2\pi\omega}{J}\right]}\right] \tag{35}$$

and

$$N_y = (-1)^n N_0 \sin\alpha\ . \tag{34}$$

Because of the restrictions we have placed on the frequencies to be discussed we can eliminate ω from equations (35) and (34) and talk in terms of ω_0, n, and k. Replacing ω in equation (35) by $\frac{k}{n}\omega_0$ we obtain

$$N_x = (-1)^n\left[N_0\cos\alpha - \frac{E_0 e\,\omega_0}{2m\left[\omega_0^2\left(1-\frac{k^2}{n^2}\right)-\frac{2\pi k\omega_0}{nJ}\right]} + \frac{E_0 e\,\omega_0}{2m\left[\omega_0^2\left(1-\frac{k^2}{n^2}\right)+\frac{2\pi k\omega_0}{nJ}\right]}\right]$$

$$+(-1)^k\left[\frac{E_0 e\,\omega_0}{2m\left[\omega_0^2\left(1-\frac{k^2}{n^2}\right)+\frac{2\pi k\omega_0}{nJ}\right]} - \frac{E_0 e\,\omega_0}{2m\left[\omega_0^2\left(1-\frac{k^2}{n^2}\right)-\frac{2\pi k\omega_0}{nJ}\right]}\right]. \tag{36}$$

Let us denote equation (36) as follows:

$$N_x = (-1)^n N_0\cos\alpha + \beta(n,k)\,. \tag{37}$$

In equation (36) the expression

$$\frac{E_0 e \omega_0}{2m\left[\omega_0^2\left(1-\frac{k^2}{n^2}\right)+\frac{2\pi k\omega_0}{n\mathcal{T}}\right]} - \frac{E_0 e \omega_0}{2m\left[\omega_0^2\left(1-\frac{k^2}{n^2}\right)-\frac{2\pi k\omega_0}{n\mathcal{T}}\right]}$$

can be written as

$$\frac{E_0 e \omega_0}{2m}\left[\frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right)+\frac{2\pi k\omega_0}{n\mathcal{T}}} - \frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right)-\frac{2\pi k\omega_0}{n\mathcal{T}}}\right].$$

We wish to investigate the behavior of this last bracketed expression as a function of k because it will tell us how β varies as a function of frequency.

$$\frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right)+\frac{2\pi k\omega_0}{n\mathcal{T}}} - \frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right)-\frac{2\pi k\omega_0}{n\mathcal{T}}} = \frac{-\frac{4\pi k\omega_0}{n\mathcal{T}}}{\omega_0^4\left(1-\frac{k^2}{n^2}\right)^2-\frac{4\pi^2k^2\omega_0^2}{n^2\mathcal{T}^2}}$$

Since this expression has a value $\frac{\mathcal{T}}{\pi\omega_0}$ for k = n let us divide the expression by $\frac{\mathcal{T}}{\pi\omega_0}$ to obtain an expression which will give us the ratio of the value of the original expression to its value at k = n. Let this new expression have a name, say, F(n,k).

$$F(n,k) = \frac{-\frac{4\pi^2 k\omega_0^2}{n\mathcal{T}^2}}{\omega_0^4\left(1-\frac{k^2}{n^2}\right)^2-\frac{4\pi^2k^2\omega_0^2}{n^2\mathcal{T}^2}} = \frac{1}{\frac{k}{n} - \frac{\omega_0^2\mathcal{T}^2\left(\frac{n}{k}+\frac{k^3}{n^3}-\frac{2k}{n}\right)}{4\pi^2}}$$

Now n and $\mathcal{T}$ are not independent.

$$n = \frac{2\mathcal{T}}{1/f_0} = 2f_0\mathcal{T} = \frac{2\omega_0\mathcal{T}}{2\pi} \qquad \text{and so } \mathcal{T} = \frac{n\pi}{\omega_0}.$$

Hence F(n,k) =

$$\frac{1}{\frac{k}{n} - \frac{1}{4}\left(\frac{n^3}{k}+\frac{k^3}{n}-2nk\right)}. \qquad (38)$$

Let us change the form of equation (36).

$$v_x = (-1)^n v_0 \cos\alpha + (-1)^n \left\{ \frac{E_0 e \omega_0}{2m} \left[\frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right) + \frac{2\pi k \omega_0}{n \tau}} \right.\right.$$

$$\left.\left. - \frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right) - \frac{2\pi k \omega_0}{n \tau}} \right] \right\} + (-1)^k \left\{ \left[\frac{E_0 e \omega_0}{2m} \right] \left[\frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right) + \frac{2\pi k \omega_0}{n \tau}} \right.\right.$$

$$\left.\left. - \frac{1}{\omega_0^2\left(1-\frac{k^2}{n^2}\right) - \frac{2\pi k \omega_0}{n \tau}} \right] \right\}$$

$$v_x = (-1)^n v_0 \cos\alpha + (-1)^n \frac{E_0 e \omega_0}{2m} \frac{\tau}{\pi \omega_0} F(n,k)$$

$$+ (-1)^k \frac{E_0 e \omega_0}{2m} \frac{\tau}{\pi \omega_0} F(n,k)$$

$$v_x = (-1)^n v_0 \cos\alpha + \left[(-1)^n + (-1)^k\right] \frac{E_0 e \tau}{2\pi m} F(n,k) \tag{39}$$

In equation (37) we denoted v_x as

$$v_x = (-1)^n v_0 \cos\alpha + \beta(n,k).$$

Hence by equation (39)

$$\beta(n,k) = \left[(-1)^n + (-1)^k\right] \frac{E_0 e \tau}{2\pi m} F(n,k). \tag{40}$$

Equation (34) is

$$v_y = (-1)^n v_0 \sin\alpha.$$

Equations (34), (37), (38), and (40) constitute the solutions to the differential equations in final form.

Chapter III

PRELIMINARY CALCULATIONS

3.1 Calculation of Δr

Let us begin consideration of the important special case where n = k (or $\omega = \omega_0$). A few cases where $\frac{\omega}{2\pi}$ is not equal to the natural frequency will be considered later. For simplicity let n be an even number. As noted previously this introduces an error in the final speed which corresponds to energy gained or lost in a fraction of a cycle and hence should amount to a fraction of one per cent. Equations (34), (37), (38), and (40) give

$$v_x = v_0 \cos\alpha + \frac{E_0 e T}{\pi m} \tag{41}$$

and

$$v_y = v_0 \sin\alpha \quad . \tag{42}$$

The resultant speed is

$$v = \sqrt{v_x^2 + v_y^2} \quad . \tag{43}$$

Hence the resultant radius is

$$r = \frac{1}{\omega_0}\sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{v_x}{\omega_0}\right)^2 + \left(\frac{v_y}{\omega_0}\right)^2} \quad . \tag{44}$$

The change in radius is

$$\Delta r = r - r_0 = \sqrt{\left(\frac{v_x}{\omega_0}\right)^2 + \left(\frac{v_y}{\omega_0}\right)^2} - r_0 \quad . \tag{45}$$

Consider an example:

Let $r_0 = (v_0/\omega_0) = 10^{-5}$ meter

$\alpha = 45^\circ$

$\beta = 6\pi \times 10^4$ meter/second

$\omega_0 = 2\pi \times 3 \times 10^9$ radians/second

Then $\dfrac{v_x}{\omega_0} = \dfrac{\omega_0 \times 10^{-5}\cos 45^\circ + 6\pi \times 10^{4}}{\omega_0} = 10^{-5}\cos 45^\circ + 10^{-5}.$

$$\frac{v_y}{\omega_0} = \frac{\omega_0 \times 10^{-5}\sin 45^\circ}{\omega_0} = 10^{-5}\sin 45^\circ$$

$$\frac{v_x}{\omega_0} = 1.707 \times 10^{-5} \text{ meter}$$

$$\frac{v_y}{\omega_0} = .707 \times 10^{-5} \text{ meter}$$

$$\Delta r = \sqrt{\left(\frac{v_x}{\omega_0}\right)^2 + \left(\frac{v_y}{\omega_0}\right)^2} - r_0 = \sqrt{(1.707\times10^{-5})^2 + (.707\times10^{-5})^2} - 10^{-5}$$

$$\Delta r = (1.85 - 1)10^{-5} = .85 \times 10^{-5} \text{ meter}$$

Let this be interpreted to be one point on a curve of $\Delta r(r_0, \alpha, \beta)$ versus r_0 for fixed values of α and β.

It may be for certain combinations of r_0, α, and β that Δr is nearly independent of r_0. That this is so will be shown as follows:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(v_0\cos\alpha + \beta)^2 + (v_0\sin\alpha)^2},$$

$$v = \sqrt{v_0^2 + 2\beta v_0\cos\alpha + \beta^2}.$$

If $\beta \ll v_0\cos\alpha$ then

$$v \approx \sqrt{v_0^2 + 2\beta v_0\cos\alpha}$$

The quantity on the right hand side can be expanded by the binomial theorem.

$$v \approx v_0 + \frac{2\beta v_0\cos\alpha}{2v_0} + \ldots$$

Hence

$$\Delta v \approx \beta\cos\alpha$$

and

$$\Delta r \approx \frac{\beta\cos\alpha}{\omega_0} = \frac{E_0 e T\cos\alpha}{\pi m\omega_0} \qquad (46)$$

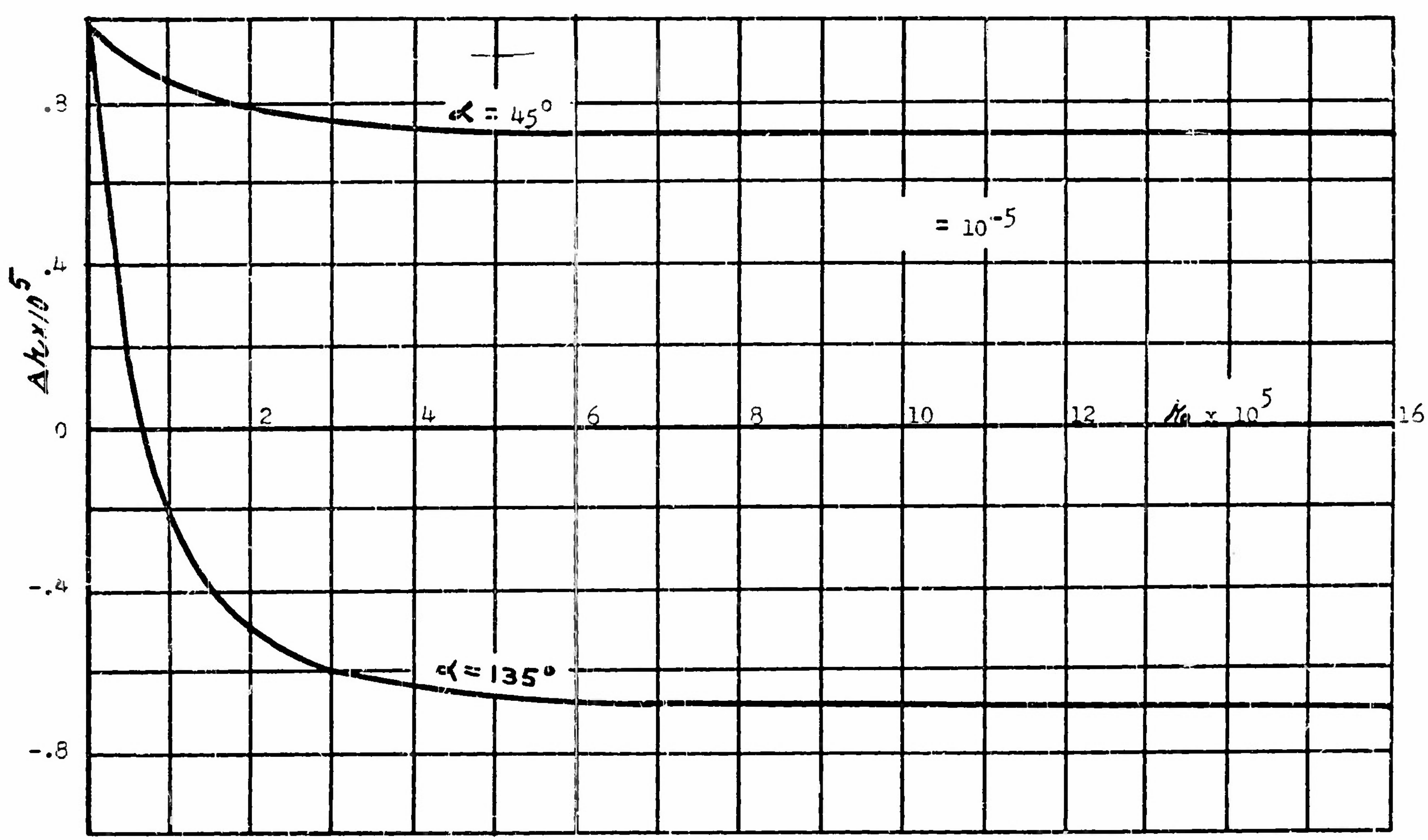


Figure 3 -- Typical Curves of Δh versus h.

3.2 Calculation of δ_i

We have established a numerical approximation to the signal current at the grid, namely,

$$\Delta I_b = \sum_{i=0}^{\ell} \frac{A_i}{A_T}\,\delta_i \; . \tag{8}$$

In setting up this formula we considered the cathode current approaching the grid openings to be unity. Hence the signal current at the grid given by this formula will be some fraction of this unit current. In section 1.5 we defined A_i, A_T, and δ_i. We will calculate a set of δ_i corresponding to some input signal power. A set of A_i will be easy to calculate. From these we can obtain one point of a curve of signal current versus input signal power.

The equation for $\Delta\Lambda$ is

$$\Delta\Lambda = \sqrt{\left(\frac{\Lambda_0\cos\alpha+\beta}{\omega_0}\right)^2+\left(\frac{\Lambda_0\sin\alpha}{\omega_0}\right)^2} - \Lambda_0 \; . \tag{45}$$

$$\Delta\Lambda = \sqrt{\left(\Lambda_0\cos\alpha+\frac{\beta}{\omega_0}\right)^2+\left(\Lambda_0\sin\alpha\right)^2} - \Lambda_0 \tag{47}$$

From this last equation we see that $\Delta\Lambda$ is a function of Λ_0, α, and β/ω_0. We will hold ω_0 constant and thus consider operation at a fixed magnetic flux density. Electrons leaving the cathode will have various values of Λ_0, as given by the distribution function g(Λ_0). As the electrons are emitted from the cathode the sidewise component of velocity may have any direction. Each direction corresponds to a value of α. Thus the electrons are randomly distributed with respect to α. The remaining parameter is β/ω_0. The relationship between β and input power (and hence between β/ω_0 and input power) will be shown next.

From the solution of the differential equations

$$E_0 = \frac{\pi m \beta}{e \mathcal{T}} \quad . \tag{48}$$

Input power and E_0 are related as follows:

$$P = K_1 E_0^{\,2} . \tag{49}$$

It follows that input power is directly proportional to $(\beta/\omega_0)^2$. The constant, K_1, depends upon guide width, height, and signal frequency. Let us therefore plot signal current against β/ω_0. For any special case, then, β/ω_0 can be expressed in terms of input power.

Since the electrons are randomly distributed with respect to α we will divide the initial electrons into a number of parts (ten), each part having an average value for α. We choose values of 9°, 27°, 45°, 63°, 81°, 99°, 117°, 135°, 153°, and 171°. It will not be necessary to use values between 180° and 360° since $\Delta\Lambda$ for $\alpha = \theta$ is the same as $\Delta\Lambda$ for $\alpha = -\theta$. Each of these parts will contain 1/10 of the electrons.

We next construct a family of curves of $\Delta\Lambda$ versus Λ_0 for β/ω_0 fixed and α taking on the values given above. A sample calculation has been previously given. The choice of a suitable value for β/ω_0 is a matter of trial and error. For one choice consider $\beta/\omega_0 = 10^{-5}$ ($\beta = 6\pi \times 10^4$, $\omega_0 = 2\pi \times 3 \times 10^9$). The way in which these curves can be used will soon be explained.

First let us consider some figures in regard to the initial distribution of radii. Since

$$g(r_0) = \frac{m\omega_0^2 r_0}{KT} \epsilon^{-\frac{m\omega_0^2}{2KT} r_0^2} \tag{5}$$

it follows that

$$\int_{r_1}^{r_2} g(r_0)\, dr_0 = \epsilon^{-\frac{m\omega_0^2}{2KT} r_1^2} - \epsilon^{-\frac{m\omega_0^2}{2KT} r_2^2} . \tag{50}$$

This last expression gives the portion of the electrons whose radii lie between r_1 and r_2 . Suppose T = 850°C, a typical value for a cathode. Further suppose that $r_1 = .5 \times 10^{-5}$ meter and $r_2 = .75 \times 10^{-5}$. Then the portion of the electrons whose radii lie between these limits has a value:

$$\epsilon^{-.261} - \epsilon^{-.5873} = .7695 - .5550 = .2145 .$$

Thus 21.45 per cent of the electrons fall in this category. We construct the following table of electrons whose radii fall in groups of width .25 x 10^{-5} meter.

Table 1 -- Initial Distribution of Electrons

$r_0 \times 10^5$ center value	.125	.375	.625	.875	1.125	1.375	1.625
Decimal part	.0634	.1671	.2145	.2045	.1560	.0995	.0544
$r_0 \times 10^5$ center value	1.875	2.125	2.375	2.625	2.875	3.125	
Decimal part	.0253	.0103	.0036	.0010	.0003	.0001	

We are now ready to begin use of the curves Δr versus r_0. Consider an example: from the table just constructed 21.45 per cent of the electrons belong to the group centered on $r_0 = .625 \times 10^{-5}$ meter. Of these electrons 1/10 correspond to $\alpha = 9^\circ$, 1/10 to $\alpha = 27^\circ$, etc. Consider the 1/10 for which $\alpha = 45^\circ$. Looking at the curve of Δr versus r_0 for $\alpha = 45^\circ$ we see that if $r_0 = .625 \times 10^{-5}$ meter then $\Delta r = .88 \times 10^{-5}$ meter. Let us begin the construction of a table showing the distribution of final radii.

Table 2 -- Final Electrons with Radii in Groups of Width $.5 \times 10^{-5}$ Meter (Except the First Group)

0 to $.25 \times 10^{-5}$	$(.25$ to $.75) \times 10^{-5}$	$(.75$ to $1.25) \times 10^{-5}$	$(1.25$ to $1.75) \times 10^{-5}$	ETC.
.	.	.	.02145	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	

Since for our example r_o = .625 x 10^{-5} and Δr = .88 x 10^{-5} meter the final radius, r_f = .625 x 10^{-5} + .88 x 10^{-5} = 1.505 x 10^{-5}. The quantity of initial electrons, namely 1/10 x .2145, which originally had radii lying between .5 x 10^{-5} meter and .75 x 10^{-5} meter have had their radii increased after crossing the wave guide and now fall in a new group as shown by the table. Initial electrons which don't clearly fall into just one of the final groups (suppose final r = 1.25 x 10^{-5} meter) can be divided between adjacent groups. When this work is carried out the following results will be obtained:

Table 3 -- The Results from Table 2

Final groups in terms of radius	Portion of total electrons contained
0 to .25 x 10^{-5}	.0664
(.25 to .75) x 10^{-5}	.1753
(.75 to 1.25) x 10^{-5}	.2671
(1.25 to 1.75) x 10^{-5}	.2827
(1.75 to 2.25) x 10^{-5}	.1491
(2.25 to 2.75) x 10^{-5}	.0499
(2.75 to 3.75) x 10^{-5}	.0093
(3.25 to 3.75) x 10^{-5}	.0011
(3.75 to 4.25) x 10^{-5}	.0001

From this table we can construct another table which shows the portion of the electrons which have radii exceeding any given r_i .

From the bottom line of the above table we see that a portion .0001 have radii exceeding 3.75×10^{-5} meter. By adding in the electrons from the next group above this one we see that a portion .0001 + .0011 = .0012 have radii exceeding 3.25×10^{-5} meter, etc.

Table 4 -- Final Electrons with Radii Exceeding r_i
($\beta/\omega_0 = 10^{-5}$)

i	r_i	Decimal part
0	$.25 \times 10^{-5}$	.9346
1	$.75 \times 10^{-5}$	.7593
2	1.25×10^{-5}	.4922
3	1.75 x	.2095
4	2.25	.0603
5	2.75	.0105
6	3.25	.0012
7	3.75	.0001

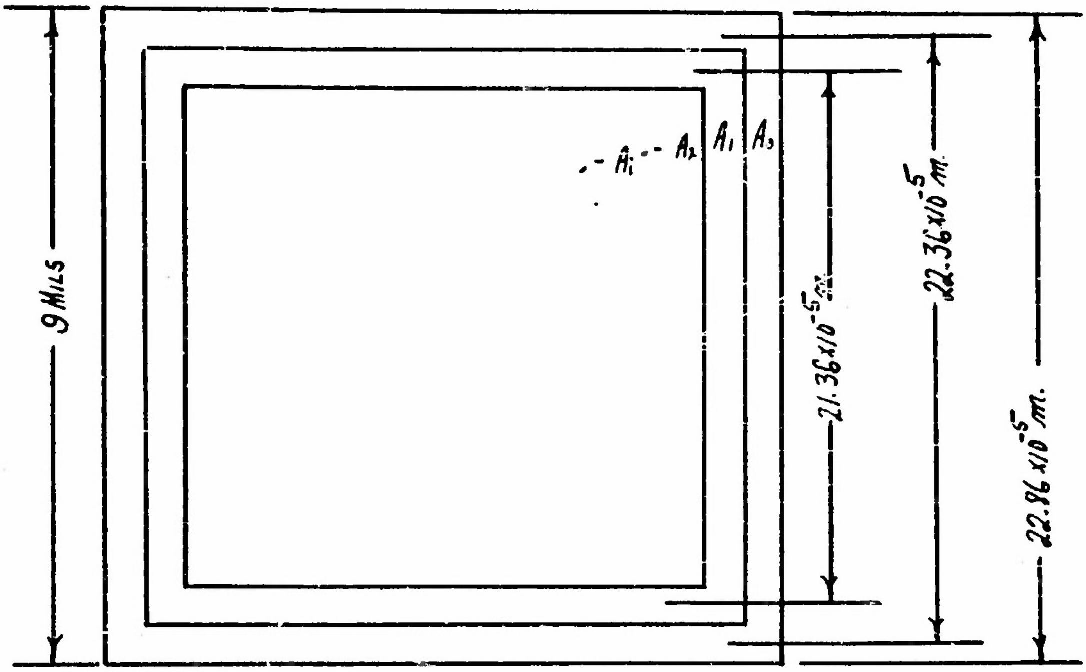
We have already established that the portion of initial electrons with radii exceeding any given r_i is given by $\epsilon^{-\frac{m\omega_0}{2KT} r_i^2}$. We now calculate a set of δ_i by the method outlined in section 1.5.

Table 5 -- A Set of δ_i

$$\delta_0 = .9346 - \epsilon^{-\frac{m\omega_0^2}{2KT}(.25\times10^{-5})^2} = .9346 - .9366 = -.002$$
$$\delta_1 = .7593 - .5550 = .2043$$
$$\delta_2 = .4922 - .1945 = .2977$$
$$\delta_3 = .2095 - .0406 = .1689$$
$$\delta_4 = .0603 - .0050 = .0553$$
$$\delta_5 = .0105 - .0004 = .0101$$
$$\delta_6 = .0012 - 0 = .0012$$
$$\delta_7 = .0001 - 0 = .0001$$

3.3 Calculation of A_i

Consider a grid with openings 9 mils square. The following diagram should explain itself:



A_t = total area = $(22.86 \times 10^{-5})^2 = 522.56 \times 10^{-10}$ sq. meter

Table 6 -- A Set of A_i

i	0	1	2	3	4	5	6
$A_i \times 10^{10}$	44.72	42.72	40.72	38.72	36.72	34.72	32.72
i	7	8	9	10	11	12	13
$A_i \times 10^{10}$	30.72	28.72	26.72	24.72	22.72	20.72	18.72
i	14	15	16	17	18	19	20
$A_i \times 10^{10}$	16.72	14.72	12.72	10.72	8.72	6.72	4.72
i	21	22					
$A_i \times 10^{10}$	2.72	.74					

Chapter IV

CALCULATION OF SIGNAL CURRENTS

4.1 Calculation of Signal at the Grid

$$\Delta I_G = \frac{1}{A_T} \sum A_i \delta_i \tag{8}$$

Examples have been given for calculating A_i and δ_i. Using the results of these examples:

$$\begin{aligned}
A_0\delta_0 &= -.0894 \\
A_1\delta_1 &= 8.7277 \\
A_2\delta_2 &= 12.1223 \\
A_3\delta_3 &= 6.5398 \\
A_4\delta_4 &= 2.0306 \\
A_5\delta_5 &= .3507 \\
A_6\delta_6 &= .0393 \\
A_7\delta_7 &= .0031 \\
\hline
\sum A_i \delta_i &= 29.7241
\end{aligned}$$

$$\frac{1}{A_t}\sum_{i=0}^{7} A_i\delta_i = \frac{29.72}{522.6} = .0569$$

Thus for a grid with openings 9 mils square and a power input corresponding to $P/\omega_0 = 10^{-5}$ there will be an increase in grid current over the quiescent grid current. The amount of this increase is 5.7 per cent of the cathode current which approaches the grid openings. If the increase were expressed as a percentage of the total cathode current, then the number expressing this increase would be smaller. Thus if the grid structure had a cross sectional area ten per cent of the total area then the increase would be: $\dfrac{5.7 \times 100\%}{1 + \frac{.1}{1-.1}} = 5.13\%$.

4.2 Calculation of Signal at the Collector

The collector current plus the total grid current minus the portion of the grid current captured by the finite thickness of the grid equals unity. Thus if the grid current increases the collector current decreases by the same amount.

We will find it convenient to express the decrease in collector current as a decimal part of the quiescent collector current. This will give us a figure independent of the thickness of the grid vanes. Since the portion of the grid current which is captured by the finite thickness of the grid vanes never has a chance to reach the collector, it can not cause any percentage change in collector current.

As explained in the section 1.5 the quiescent grid current is

$$\frac{1}{A_t} \sum_{i=0}^{\rho} A_i \epsilon^{-\frac{m\omega_0^2}{2KT} r_i^2}$$

plus the current due to the finite thickness of the grid. Thus the quiescent collector current is

$$I_{co} = 1 - \frac{1}{A_t} \sum_{i=0}^{\rho} A_i \epsilon^{-\frac{m\omega_0^2}{2KT} r_i^2} . \qquad (51)$$

Evaluation of this expression for our previous example gives

$$I_{co} = .856 .$$

Thus the grid signal of .0569 represents a decrease in collector current of $\frac{.0569}{.856} \times 100\% = 6.65\%$.

4.3 Relating β/ω_o to Input Power

Suppose that the wave guide dimensions are 1/8" x 3" and that $\omega_o = 6\pi \times 10^9$ radians/second. These are values for a tube to be built at the University of Colorado. Further suppose that for an average electron the transit time corresponds to one electron volt energy. For the transit time, τ, then:

$$\tau = \frac{3.0}{39.37}\sqrt{\frac{2Ve}{m}} = \frac{3.0}{39.37}\sqrt{\frac{2 \times 1.6 \times 10^{-19}}{9.03 \times 10^{-31}}}$$

$$= 1.28 \times 10^{-7} \text{ second .}$$

For a terminated wave guide:

$$E_o = 2\sqrt{\frac{\eta_1 P}{ab\left[1-\left(\frac{f_c}{f}\right)^2\right]^{1/2}}} \; . \qquad (52)$$

f_c = cutoff freq. ($\lambda_c = 2a$), η_1 = 377 ohms, a = guide width, b = guide height, and f is the frequency of the wave.

For our case this gives:

$$P = 12.1 \times 10^{-8} E_o^{\,2} \; .$$

Also since $\frac{E_o e\tau}{\pi m} = \beta$, $E_o = \frac{\pi m \beta}{e\tau}$.

$$E_o = \frac{\pi \times 9.03 \times 10^{-31} \;\beta}{1.6 \times 10^{-19} \times 1.28 \times 10^{-7}} = 1.387 \times 10^{-4}\beta$$

Using the relations $P = 12.1 \times 10^{-8} E_\theta^{\,2}$ and $E_\theta = 1.387 \times 10^{-4} \beta$ we construct the following table:

Table 7 -- Input Power Versus β/ω_0

β meters/sec	$3\pi \times 10^4$	$6\pi \times 10^4$	$12\pi \times 10^4$	$18\pi \times 10^4$	$30\pi \times 10^4$
β/ω_0 meters	.5	1	2	3	5
P milliwatts	.0208	.0825	.331	.621	2.08

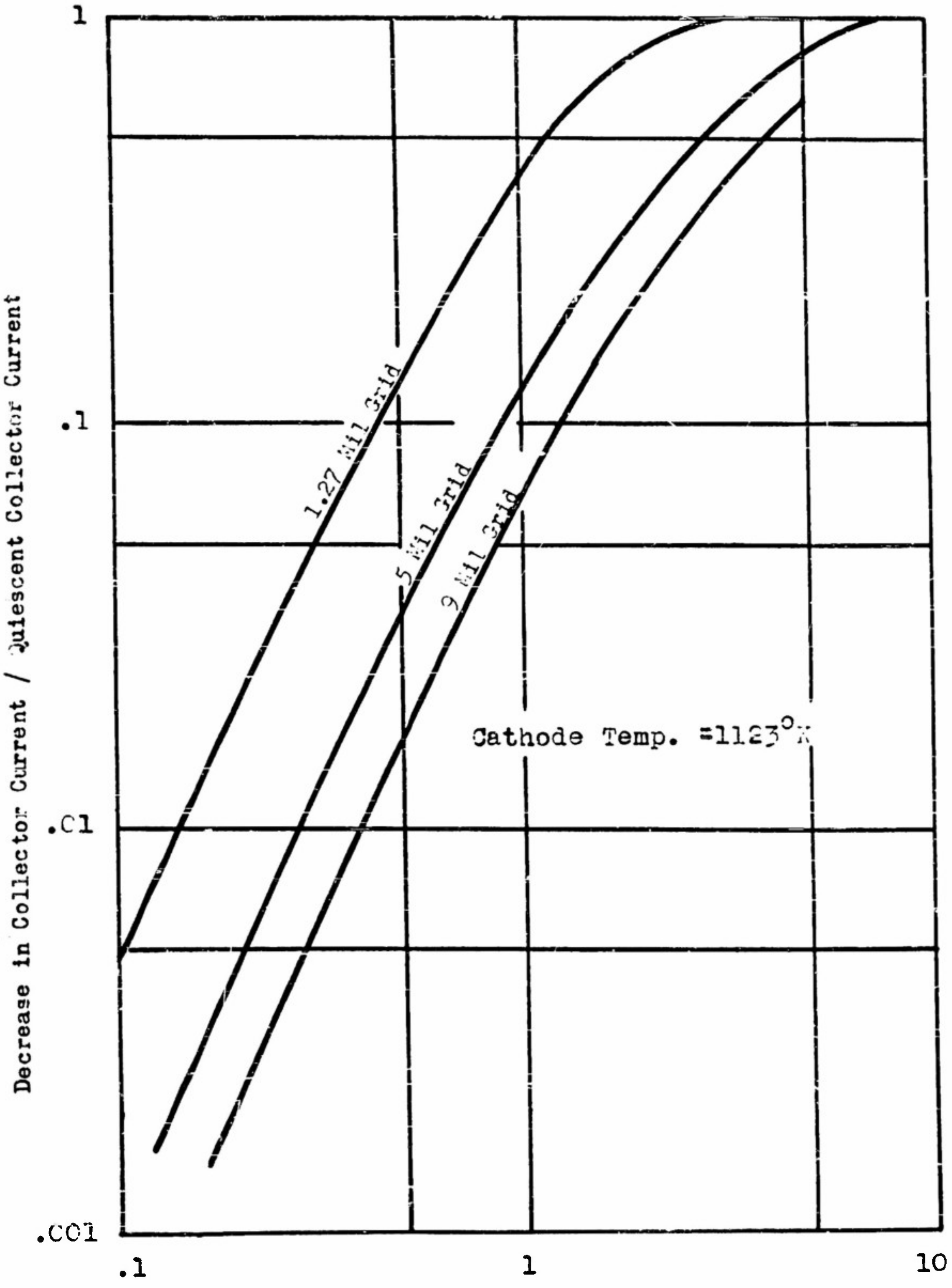


Figure 4 -- Decrease in collector current / quiescent collector current versus $\beta/\omega_0 \times 10^5$.

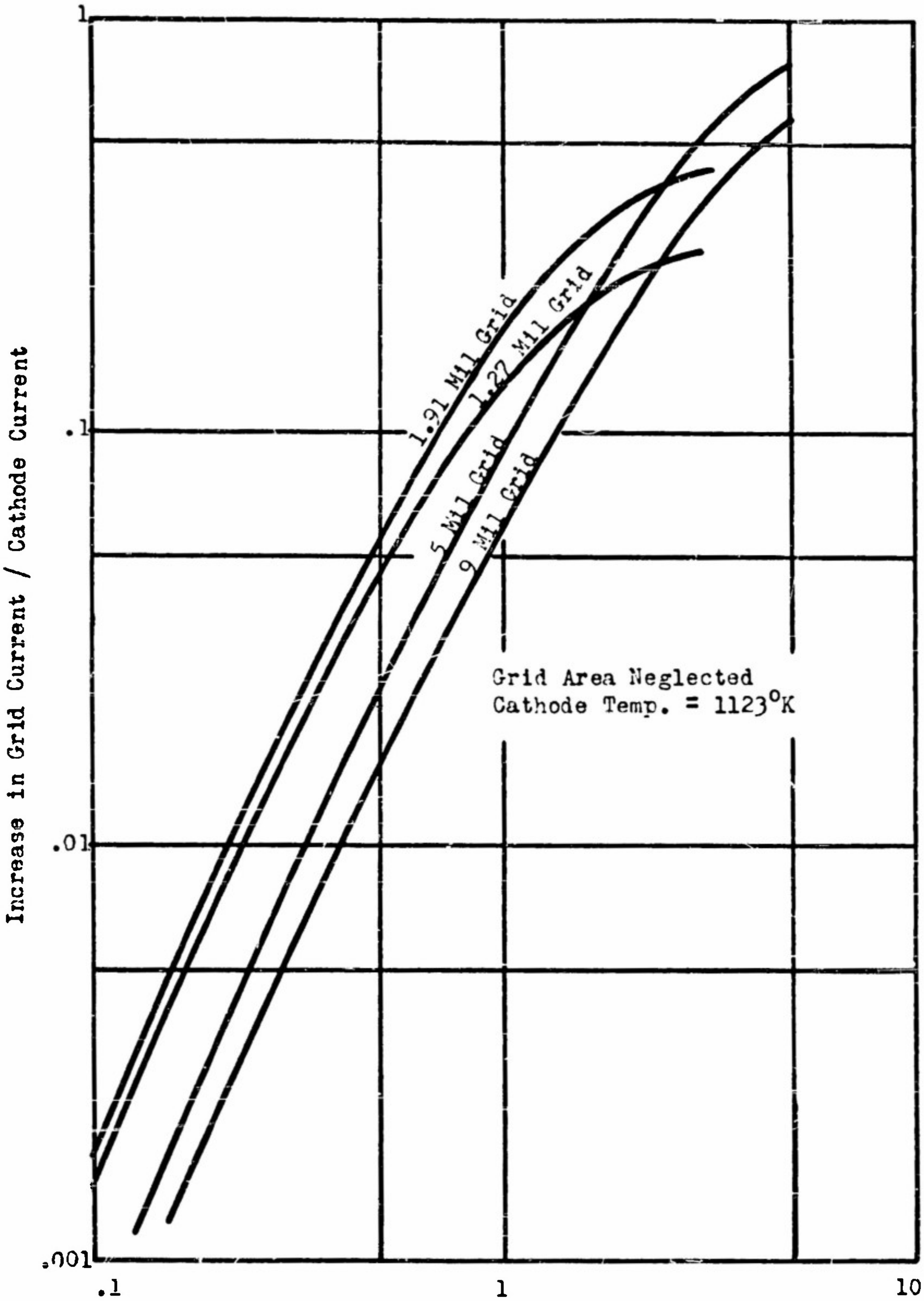


Figure 5 -- Increase in grid current / cathode current versus β/w x 10^5.

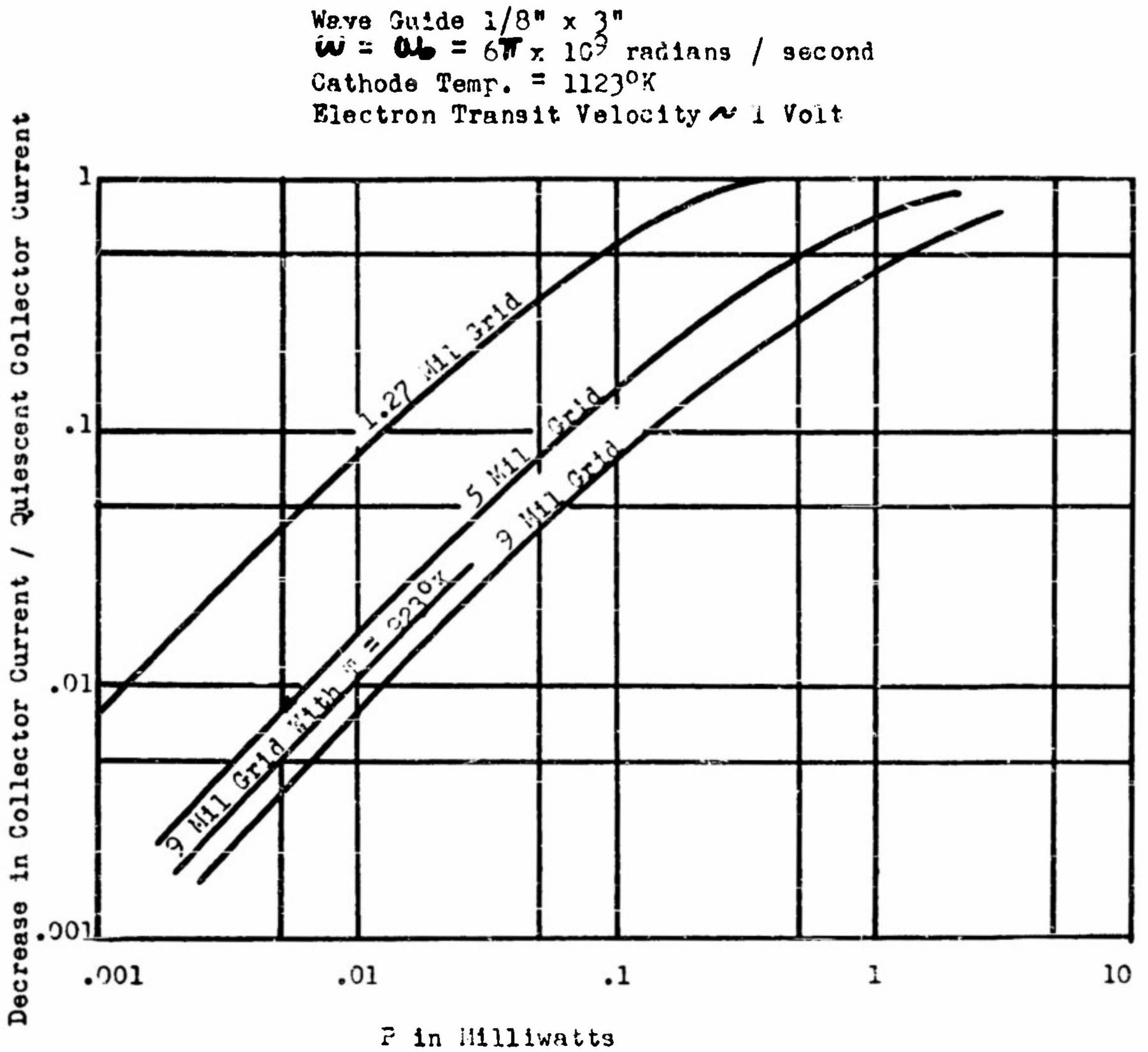


Figure 6 -- Decrease in collector current / quiescent collector current versus power in milliwatts.

Wave Guide 1/8" x 3"
$\omega = \omega_0 = 6\pi \times 10^9$ radians / second
Cathode Temp. = 1123°K
Grid Area Neglected
Electron Transit Velocity $\sim$ 1 Volt

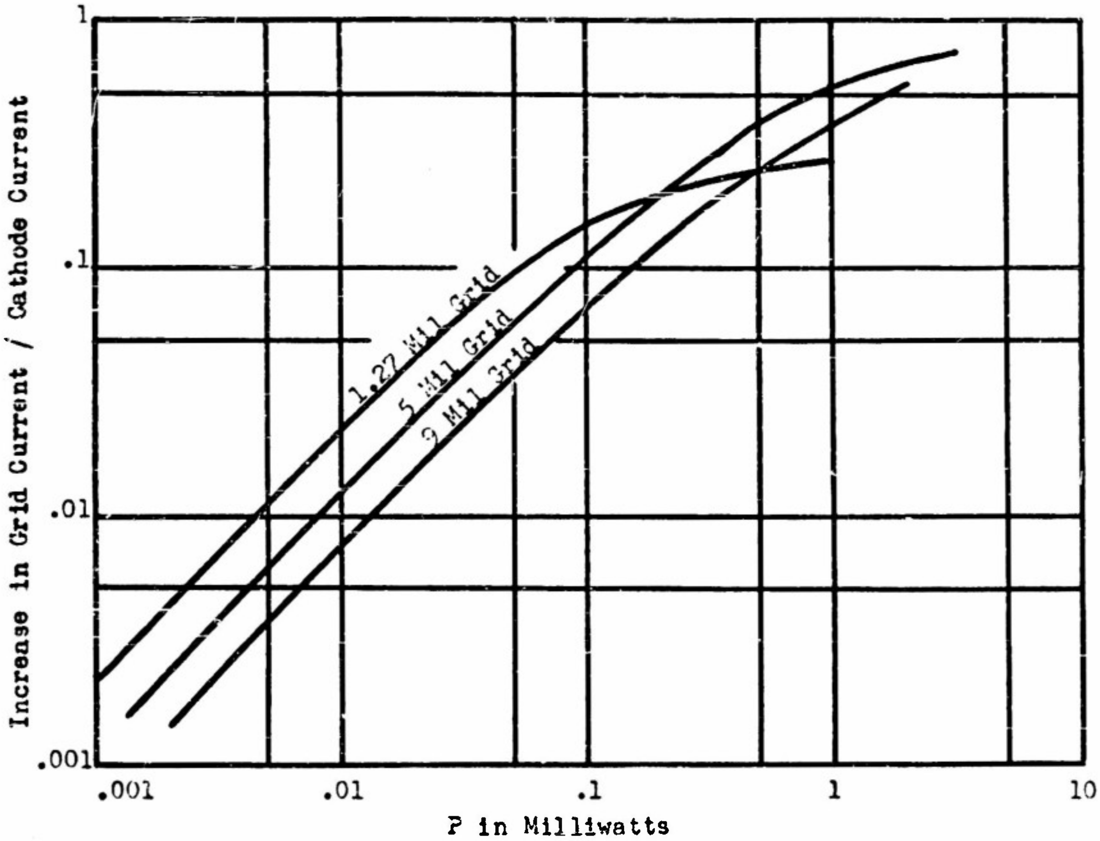


Figure 7 -- Increase in grid current / cathode current versus power in milliwatts.

Chapter V

FREQUENCY RESPONSE

5.1 Frequency Response

The solutions to the differential equations were obtained as

$$v_x = (-1)^n v_0 \cos\alpha + \beta(n,k),$$

$$v_y = (-1)^n v_0 \sin\alpha ,$$

and

$$\beta(n,k) = \frac{E_0 e \tau}{2\pi m}\left[(-1)^n + (-1)^k\right] F(n,k)$$

where

$$F(n,k) = \frac{1}{\frac{k}{n} - \frac{1}{4}\left(\frac{n^3}{k} + \frac{k^3}{n} - 2nk\right)} .$$

Let us investigate the effect of a signal differing in frequency from the natural frequency of the tube. At the natural frequency k = n. If we let k = n $\pm$ 2, say, then the frequencies being considered differ from the natural frequency, f_0, by an amount $\pm\frac{2f_0}{n}$. In order to hold the absolute value of β constant, and hence the tube output constant, E_0 must be increased as the absolute value of F(n,k) decreases.

Since input power varies as the square of E_0, P is directly proportional to

$$\left\{\frac{1}{\left[(-1)^n + (-1)^k\right] F(n,k)}\right\}^2$$

Actually in the equation $P = K_1 E_0^2$, K_1 is a function of frequency, but the variation with frequency is slow enough that it will be neglected.

Suppose n is even. Then for all odd values of k, $P = \infty$. Because of space charge effects, however, and because of the initial distribution of velocities of emission, the transit times for different electrons will vary. For an electron with a different transit time $P = \infty$ will correspond to a different set of frequencies.

For k and n both odd, or both even, $(-1)^n(-1)^k = 1$ and P is directly proportional to $\left(\frac{1}{F(n,k)}\right)^2$.

Let us construct a frequency response curve through these last points. Then, if we assume that a sharp frequency response is desired, this will be a conservative response curve.

Let us continue the special case begun in section 4.3.

Wave guide dimensions = 1/8" x 3"

$\omega_b = 6\pi \times 10^9$ radians/second

$\mathcal{T}$ = first $.707 \times 1.28 \times 10^{-7}$ second and then

1.28×10^{-7} second and then $1.414 \overset{\times 1.28}{} \times 10^{-7}$ second.

These values for $\mathcal{T}$ correspond to 2, 1, and 1/2 electron volts of energy.

Electrons with these transit times will be present in the tube being considered. The curves to be obtained may prove useful in case the tube should be modified to change the frequency response characteristic.

Evaluating $\left(\frac{1}{F(n,k)}\right)^2$ for the case

$\tau = .707 \times 1.28 \times 10^{-7}$ second the following results were obtained:

$$n = 2f_0\tau \;=\; 2 \times \frac{6\pi \times 10^9}{2\pi} \times .707 \times 1.28 \times 10^{-7} = 544$$

Table 8 -- D.B. Input Versus $(f - f_0)$

k	544	546	548	550	552	554	556	558	560	560
$\left\{\frac{1}{F(n,k)}\right\}^2$	1	3^2	15^2	35^2	63^2	99^2	143^2	195^2	255^2	323^2
D.B. INPUT = $10\log_{10}\left\{\frac{1}{F(n,k)}\right\}^2$	0	9.54	23.5	30.9	36.0	40.0	43.2	45.8	48.1	50.3
$f - f_0$ IN M.C.	0	11.03	22.06	33.09	44.12	55.15	66.18	77.21	88.24	99.27

These last results plus the values corresponding to the other two values of τ are plotted on the next page.

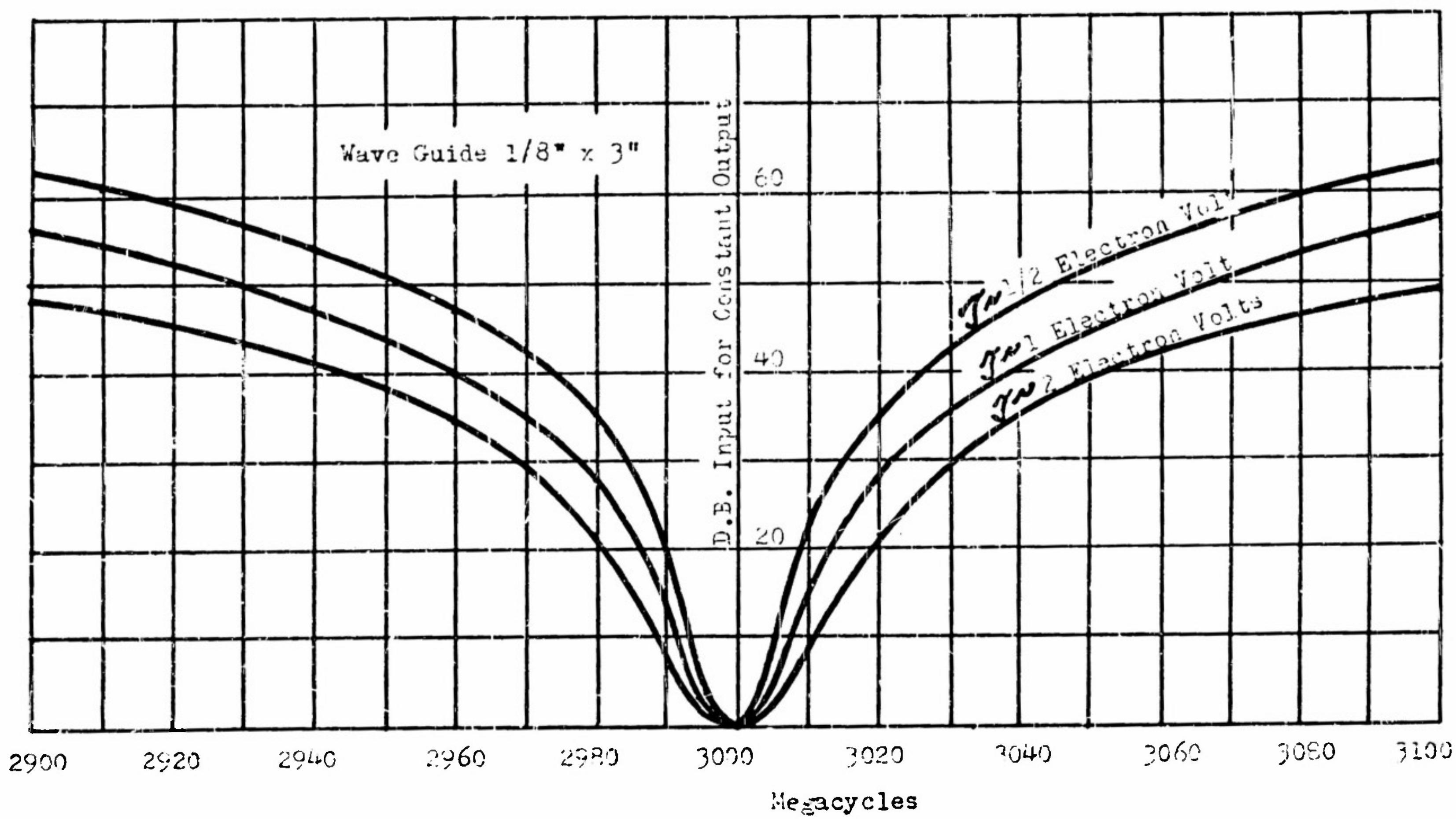


Figure 8 -- Frequency Response Curves

Chapter VI
CONCLUSIONS

6.1 Discussion of Results

It will be observed from the curves of signal output versus input that the signal output is a continuously increasing single valued function. This will be a desirable characteristic for a detector. Also the slope of the curve, collector current versus β/ω_0 , on log log paper is about two for low input power. Both of the above results are verified experimentally. Experimental results are not available to my knowledge, however, for the higher input powers. Ionization by high energy electrons may considerably alter the tube characteristics in this region.

The magnitudes given for the signal current are probably correct within a factor of two. This is indicated by data taken on a tube of somewhat different design.

It is likely that some of the factors ignored in the analysis are of some importance. Since there are actually components of electrostatic field near the grid which attract electrons to the grid, the effective grid opening is probably somewhat smaller than the actual grid opening (on the basis of this analysis). Also the wire mesh near the cathode (this name to distinguish it from the grid) changes the initial distribution of radii somewhat.

As the grid sizes are reduced the per cent change in collector current for a given input power (at low power) continues to increase for all grids here considered. However the absolute value of the change reached a maximum for a grid size of around 1.75 mils where grid area was neglected. When grid area is not neglected the optimum size for greatest absolute change will be somewhat different, depending upon how thin a grid wall can be produced for the small grid sizes.

The frequency response curves predict a sharp response for this detector, the response being sharper for slow electrons than for faster ones. By adjustment of the accelerating voltage and by design of the electron beam some control over the frequency response will be available.

6.2 Nomograms

For convenience in relating β/ω_0 to input power for a given tube two nomograms are here included.

From the equation

$$E_0 = \sqrt{\frac{P\eta_1}{ab[1-(f_c/f)^2]^{1/2}}}$$

it follows that $P = K_1 E_0^2$.

Since $E_0 = \dfrac{\pi m \beta}{e \tau}$,

$$P = \frac{K_1 \pi^2 m^2 \beta^2}{e^2 \tau^2} = K_2 \beta^2.$$

The first nomogram will relate K_1, K_2, and τ in the equation

$$K_2 = \frac{K_1 \pi^2 m^2}{e^2 \tau^2}.$$

The second nomogram will relate P, K_2, and β in the equation $P = K_2 \beta^2$.

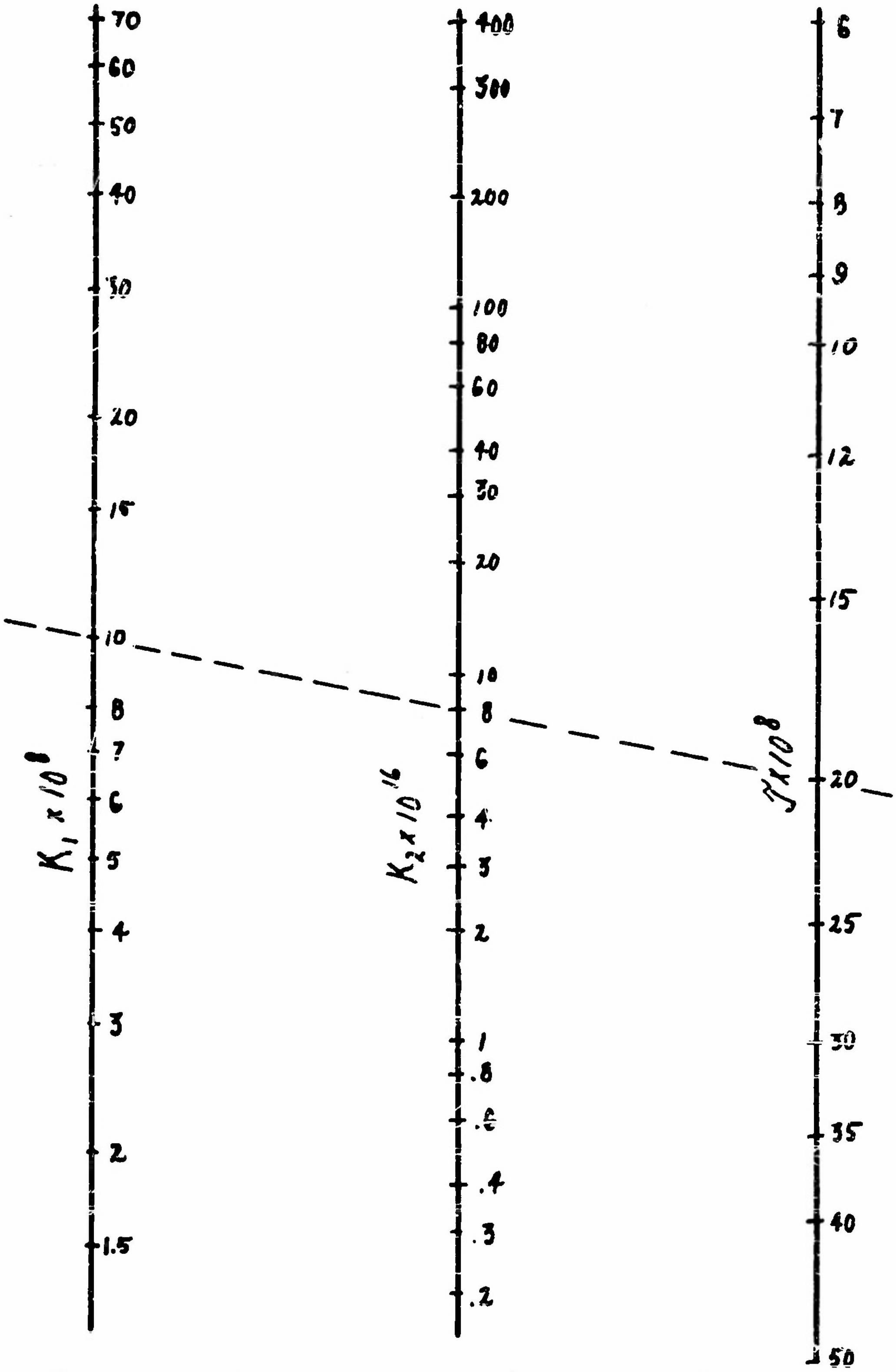


Figure 9 -- Relating K_1, X_2, and $\mathcal{J}$ in the equation

$$K_2 = \frac{2\pi m^{2}}{e^{3}\mathcal{J}^{2}}$$

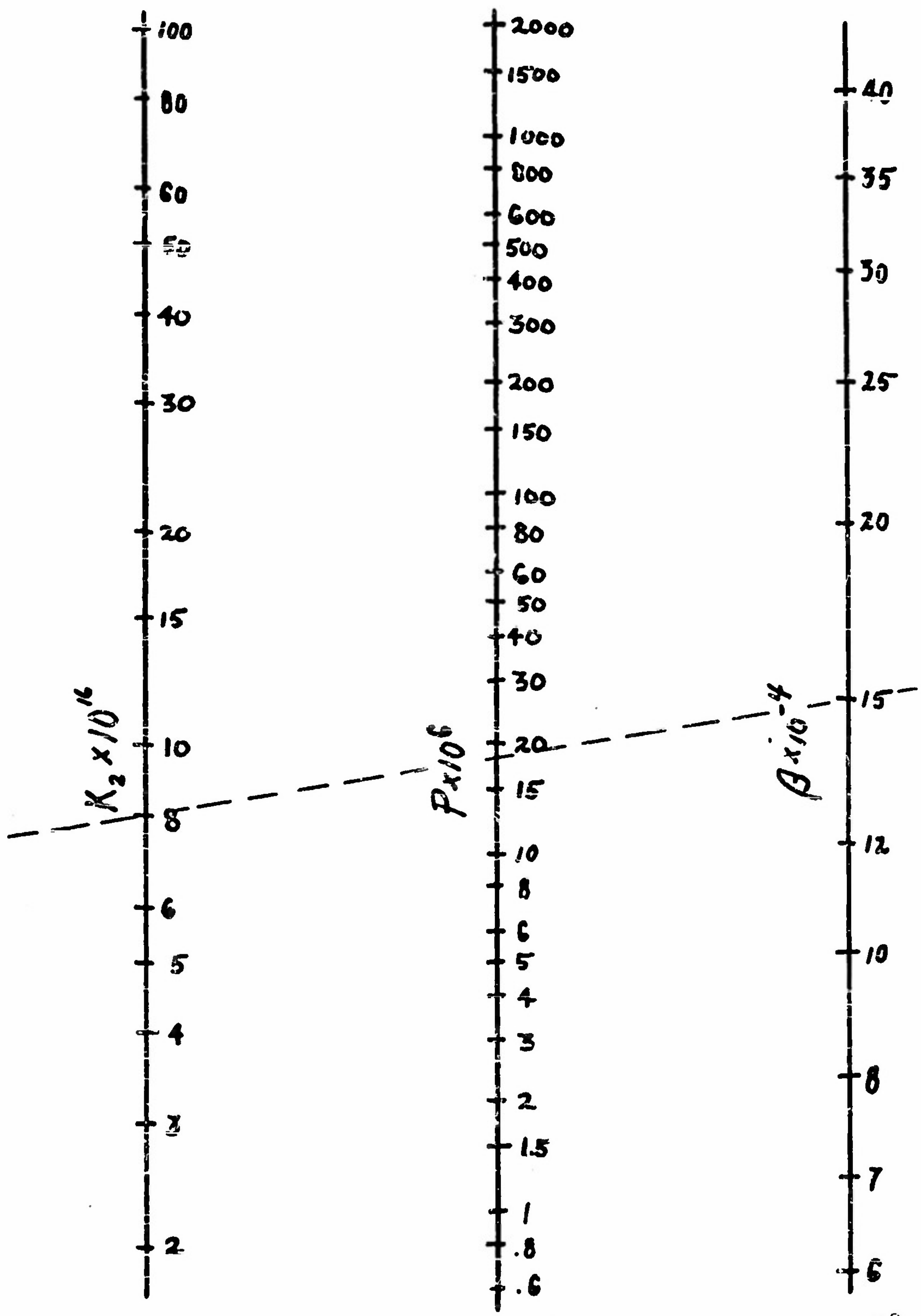


Figure 10 -- Relating P, K_2, and β in the equation $P = K_2\beta^2$

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